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Derivation of low-temperature expansions for Ising model IX. High-field polynomials for the honeycomb–triangular system

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Abstract. The derivation of high-field expansions for the honeycomb and triangular lattices is described briefly. New results are given for the high-field polynomials L_{11} and L_{12} on the triangular lattice and L_{22}, L_{23}, L_{24}, L_{25} on the honeycomb lattice. The complete codes (partial generating functions) F_{11} and F_{12} which determine the corresponding sublattice polynomials are also derived.

1. Introduction

In this paper we apply the theory and results of previous papers (Sykes *et al* 1965, 1973 a, b, c, d, e, 1975 a, b, to be referred to as I–VIII) to determine the complete codes F_{11} and F_{12} for the honeycomb–triangular code system. A general introduction is given in VII.

If we denote by S_n the number of algebraic codes of order n subject to our previous restriction (VIII, § 4) that $\alpha \geqslant 3$, then a detailed examination of the constraints (VIII, equation (4.1)) yields the result:

$$S_n = \begin{cases} \frac{1}{4}(3n^2+1) & n \text{ odd} \\ \frac{3}{4}n^2 & n \text{ even.} \end{cases} \qquad (1.1)$$

At $n = 11$ the restricted algebraic code system contains 91 codes. Some of these are trivially excluded because they correspond to powers of u below 12 in L_{11} and are therefore known to be zero from the ψ; in addition a further three primary codes ($\beta \geqslant \alpha$) are found to be non-graphical: (16, 3, 9, 4), (17, 3, 12, 2), (18, 3, 15, 0). The remaining 17 primary codes determine the complete code through the principle of balance. The secondary codes (19, 12, 0, 7) and (23, 18, 0, 5) are found to be non-graphical and the calculation fills in the remaining 54 secondary codes to give a total of 71 graphical codes in F_{11}.

At $n = 12$ the restricted algebraic code system contains 108 codes. Excluding those corresponding to powers of u below 12 in L_{12} three primary codes are found to be non-graphical: (17, 3, 9, 5), (18, 3, 12, 3), (19, 3, 15, 1). There remain 24 primary codes which determine the complete code through the principle of balance. The secondary codes (20, 12, 0, 8) and (22, 15, 0, 7) are found to be non-graphical and the calculation fills in the remaining 64 secondary codes to give a total of 88 graphical codes in F_{12}.

It is the determination of the primary codes which constitutes the practical task. We shall only describe the procedure very briefly; it is our main object to communicate the results.

2. Derivation of F_{11}

By regrouping the data of VIII the leading terms of the high-field polynomial L_{11} for the triangular lattice are obtained through u^{21}:

$$L_{11} = 24u^{12}+249u^{13}+1\,092u^{14}+1\,726u^{15}-1\,734u^{16}-21\,621u^{17}-111\,460u^{18} -136\,956u^{19}+473\,976u^{20}+2\,435\,614u^{21}+\ldots u^{33}. \quad (2.1)$$

There remain 12 coefficients to be determined. We recall that for a lattice whose high-temperature zero-field partition function and initial susceptibility expansions are known, each L_n is subject to $(n+2)$ constraints which can be used to calculate any $(n+2)$ coefficients if the remainder are given. The technique is essentially that given by Domb (1949, 1974) (see also I, § 1 and references there cited). The high-temperature expansion for the initial susceptibility is given by Sykes *et al* (1972). For L_{11} there are 13 constraints on (2.1), one more than required and this provides a check on the consistency of the data and manipulations. We give the complete polynomial in the appendix. Of the 17 primary codes in F_{11} only one, the code (22, 11, 11, 0), lies in a power of u above 21 and is therefore not known from the partial codes through F_n^{21} derived in VIII. The coefficient of u^{22} in L_{11} contains contributions from four codes:

$$(25, 20, 2, 3) \qquad (24, 17, 5, 2) \qquad (23, 14, 8, 1) \qquad (22, 11, 11, 0). \quad (2.2)$$

The first three are all secondary codes of higher class than the outstanding primary code; they can therefore be filled in by balance without resort to the last which then follows, by difference, from the coefficient of u^{22} in L_{11} already determined. The remainder of the code can then be filled in by balance. We give the complete code F_{11} in the appendix together with the honeycomb high-field polynomials L_{22}, L_{23} derived from it.

3. Derivation of F_{12}

The procedure used to derive F_{11} cannot be used to derive F_{12} as there are insufficient constraints to determine L_{12} on the triangular lattice. The coefficients of u^{22} and u^{23} would be required, although the latter could be dispensed with if the high-temperature initial susceptibility were used instead of being reserved as a check on the whole calculation. To overcome this difficulty we have exploited the special relationship between the triangular and honeycomb lattices and also provided some further configurational information.

We begin by noticing that although L_{12} for the triangular lattice cannot be calculated from the data available the polynomials L_{24} and L_{25} for the honeycomb lattice are already determined. For L_{24} it may be verified that no code in F_{12} that does not lie in F_{12}^{21} contributes to the first five coefficients which are therefore generated correctly by the partial codes derived in VIII. This follows by an inspection of the twelfth rank (II, § 2) of each code; alternatively it follows from the star-triangle substitution property (equation (2.1) of IV). We obtain

$$L_{24} = \tfrac{1}{2}z^{12}+958\tfrac{1}{2}z^{14}+67\,408\tfrac{1}{2}z^{16}+1\,085\,059\tfrac{1}{2}z^{18}-5\,401\,545\tfrac{3}{4}z^{20}+\ldots z^{72}. \quad (3.1)$$

There remain 26 coefficients and these are determined by $L_0 - L_{23}$ and the zero-field partition function and susceptibility of the honeycomb lattice. We give the completed polynomial in the appendix. The procedure may be repeated for L_{25}. From the data in F_n^{21} we obtain

$$L_{25} = 21z^{13}+6\,630z^{15}+279\,855z^{17}+2\,706\,057z^{19}-41\,340\,065z^{21}+\ldots z^{75}. \tag{3.2}$$

The remaining 27 coefficients are now determined by L_0-L_{24} and the zero-field partition function and susceptibility of the honeycomb lattice. We give the completed polynomial in the appendix.

Of the 24 primary codes which determine F_{12} only four lie in powers of u above 21. These four lie in the coefficients of u^{22}, u^{23} and u^{24} as can be seen from the scheme:

$$\begin{array}{lll}
u^{22} & (26, 20, 2, 4) & \\
 & (25, 17, 5, 3) & \\
 & (24, 14, 8, 2) & \\
 & A(23, 11, 11, 1) & \text{primary} \\
 & B(22, 8, 14, 0) & \text{primary} \\
u^{23} & (27, 22, 1, 4) & \\
 & (26, 19, 4, 3) & \\
 & (25, 16, 7, 2) & \\
 & (24, 13, 10, 1) & \\
 & C(23, 10, 13, 0) & \text{primary} \\
u^{24} & (28, 24, 0, 4) & \\
 & (27, 21, 3, 3) & \\
 & (26, 18, 6, 2) & \\
 & (25, 15, 9, 1) & \\
 & D(24, 12, 12, 0) & \text{primary.}
\end{array} \tag{3.3}$$

The counts A, B, C, D of the four primary codes must be consistent with the polynomials L_{24} and L_{25} on the honeycomb lattice; in other words they must generate the coefficients of z^{22}, z^{23}, z^{24}, z^{25} correctly, due allowance being made for the implicitly determined secondary codes. These conditions give four constraints, one of which determines A and the other three reduce to two independent constraints on B, C, and D. Any one of these latter then determines F_{12} completely. We have counted all the graphs corresponding to B and C to obtain F_{12} with one overall check on the consistency of the data. We give the complete code F_{12}, together with the triangular polynomial L_{12} derived from it, in the appendix. This latter is consistent with the high-temperature expansion for the free energy and susceptibility of the triangular lattice but this is ensured by the relationship of these expansions to the corresponding expansions on the honeycomb lattice (the star-triangle and magnetic moment transformations, equations (4.8) and (4.22) of II). The technique we have used supplements the two constraints on L_{12} (triangular) provided by the free energy and susceptibility by the one additional independent constraint: the coefficient of the *odd* power of v^{27} in the honeycomb susceptibility which does not appear in the magnetic moment result.

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Appendix

High-field polynomials L_n and complete codes F_n (for earlier terms see I, appendix and III, equations (2.2), (2.7) and appendix).

Triangular lattice

$$\begin{aligned} L_{11} = {} & 24u^{12}+249u^{13}+1\,092u^{14}+1\,726u^{15}-1\,734u^{16}-21\,621u^{17}-111\,460u^{18} \\ & -136\,956u^{19}+473\,976u^{20}+2\,435\,614u^{21}+2\,166\,684u^{22}-2\,717\,664u^{23} \\ & -57\,651\,706u^{24}-49\,830\,015u^{25}+470\,573\,262u^{26}+620\,712\,959u^{27} \\ & -5\,740\,450\,992u^{28}+12\,308\,436\,150u^{29}-13\,672\,134\,480u^{30} \\ & +8\,680\,168\,152u^{31}-3\,004\,346\,190u^{32}+442\,432\,930\tfrac{1}{11}u^{33} \end{aligned}$$

$$\begin{aligned} L_{12} = {} & 2u^{12}+117u^{13}+702u^{14}+2\,539u^{15}+3\,567u^{16}-9\,036u^{17}-65\,829u^{18}-224\,955u^{19} \\ & -466\,116u^{20}+1\,264\,848u^{21}+6\,726\,904\tfrac{1}{2}u^{22}+9\,841\,791u^{23} \\ & -23\,735\,909u^{24}-85\,016\,877u^{25}-300\,005\,784u^{26}+729\,419\,806\tfrac{2}{3}u^{27} \\ & +3\,248\,161\,023\tfrac{3}{4}u^{28}-4\,844\,214\,969u^{29}-30\,167\,516\,786\tfrac{1}{2}u^{30} \\ & +116\,686\,016\,019u^{31}-191\,760\,945\,741u^{32}+179\,792\,618\,236u^{33} \\ & -100\,311\,816\,538\tfrac{1}{2}u^{34}+31\,215\,497\,907u^{35}-4\,195\,534\,922u^{36} \end{aligned}$$

Honeycomb lattice

$$\begin{aligned} L_{22} = {} & 7z^{12}+2\,593\tfrac{1}{2}z^{14}+102\,825z^{16}+708\,809\tfrac{1}{2}z^{18}-19\,734\,007\tfrac{1}{2}z^{20}-287\,893\,480\tfrac{1}{2}z^{22} \\ & +2\,795\,027\,460\tfrac{1}{2}z^{24}+49\,747\,385\,301z^{26}-624\,043\,807\,287z^{28} \\ & -2\,593\,361\,625\,675z^{30}+106\,301\,589\,462\,448\tfrac{1}{2}z^{32}-1\,123\,511\,428\,929\,160\tfrac{1}{2}z^{34} \\ & +7\,269\,692\,214\,841\,041z^{36}-33\,565\,196\,026\,246\,911z^{38} \\ & +117\,744\,740\,909\,750\,890\tfrac{1}{2}z^{40}-324\,312\,778\,629\,323\,966\tfrac{1}{2}z^{42} \\ & +714\,787\,447\,666\,934\,928\tfrac{3}{22}z^{44}-1\,274\,264\,918\,977\,425\,250\tfrac{1}{2}z^{46} \\ & +1\,846\,874\,105\,394\,891\,727\tfrac{1}{2}z^{48}-2\,177\,403\,937\,094\,645\,532z^{50} \\ & +2\,080\,190\,064\,953\,241\,667\tfrac{1}{2}z^{52}-1\,596\,597\,474\,915\,537\,593z^{54} \\ & +970\,271\,330\,535\,085\,816\tfrac{1}{2}z^{56}-456\,274\,194\,630\,192\,639z^{58} \\ & +160\,116\,776\,680\,274\,387z^{60}-39\,466\,893\,319\,702\,027\tfrac{1}{2}z^{62} \\ & +6\,095\,410\,083\,742\,375\tfrac{1}{2}z^{64}-443\,799\,836\,388\,748\tfrac{15}{22}z^{66} \end{aligned}$$

$$L_{23} = 105z^{13} + 14\,264z^{15} + 357\,888z^{17} + 487\,428z^{19} - 83\,441\,107z^{21} - 677\,330\,547z^{23}$$
$$+ 13\,539\,400\,782z^{25} + 118\,369\,911\,810z^{27} - 2\,541\,176\,995\,590z^{29}$$
$$+ 163\,094\,873\,400z^{31} + 326\,644\,059\,249\,089z^{33} - 4\,187\,255\,733\,477\,087z^{35}$$
$$+ 30\,574\,807\,417\,758\,588z^{37} - 156\,369\,544\,197\,662\,310z^{39}$$
$$+ 604\,160\,786\,802\,794\,955z^{41} - 1\,832\,065\,120\,436\,948\,790z^{43}$$
$$+ 4\,456\,845\,491\,797\,102\,710z^{45} - 8\,811\,360\,010\,582\,030\,857z^{47}$$
$$+ 14\,258\,867\,183\,787\,609\,072z^{49} - 18\,937\,812\,984\,255\,162\,143z^{51}$$
$$+ 20\,619\,047\,663\,785\,062\,696z^{53} - 18\,310\,331\,354\,783\,877\,312z^{55}$$
$$+ 13\,136\,045\,832\,504\,193\,385z^{57} - 7\,497\,672\,230\,339\,329\,158z^{59}$$
$$+ 3\,325\,456\,001\,392\,634\,178z^{61} - 1\,104\,765\,095\,511\,466\,578z^{63}$$
$$+ 258\,650\,672\,223\,492\,633z^{65} - 38\,055\,990\,561\,230\,409z^{67}$$
$$+ 2\,646\,749\,564\,008\,905\tfrac{1}{23}z^{69}$$

$$L_{24} = \tfrac{1}{2}z^{12} + 958\tfrac{1}{2}z^{14} + 67\,408\tfrac{1}{2}z^{16} + 1\,085\,059\tfrac{1}{2}z^{18} - 5\,401\,545\tfrac{3}{4}z^{20} - 309\,280\,827z^{22}$$
$$- 1\,014\,719\,584\tfrac{7}{8}z^{24} + 55\,171\,821\,765z^{26} + 186\,987\,194\,212\tfrac{1}{2}z^{28}$$
$$- 9\,324\,780\,659\,902\tfrac{1}{2}z^{30} + 32\,155\,274\,758\,264\tfrac{7}{8}z^{32} + 919\,354\,262\,073\,286\tfrac{1}{2}z^{34}$$
$$- 15\,034\,792\,691\,560\,081\tfrac{1}{12}z^{36} + 124\,776\,008\,614\,232\,514z^{38}$$
$$- 706\,944\,446\,942\,855\,882\tfrac{5}{8}z^{40} + 3\,000\,903\,758\,162\,225\,208z^{42}$$
$$- 9\,978\,133\,770\,155\,924\,607z^{44} + 26\,649\,840\,294\,415\,627\,315\tfrac{1}{2}z^{46}$$
$$- 58\,044\,299\,211\,461\,080\,344\tfrac{1}{8}z^{48} + 104\,023\,123\,579\,945\,992\,112\tfrac{1}{2}z^{50}$$
$$- 154\,088\,330\,201\,790\,844\,602\tfrac{3}{4}z^{52} + 188\,829\,158\,075\,628\,906\,882\tfrac{1}{3}z^{54}$$
$$- 190\,962\,048\,625\,727\,144\,650\tfrac{7}{8}z^{56} + 158\,404\,160\,268\,154\,308\,909z^{58}$$
$$- 106\,670\,309\,153\,023\,369\,591\tfrac{1}{2}z^{60} + 57\,393\,499\,541\,432\,031\,346\tfrac{1}{2}z^{62}$$
$$- 24\,086\,632\,977\,121\,786\,795\tfrac{7}{8}z^{64} + 7\,596\,876\,110\,398\,265\,641\tfrac{1}{2}z^{66}$$
$$- 1\,693\,635\,680\,210\,018\,114\tfrac{1}{4}z^{68} + 237\,927\,261\,686\,657\,268z^{70}$$
$$- 15\,838\,464\,900\,601\,623\tfrac{1}{24}z^{72}$$

$$L_{25} = 21z^{13} + 6\,630z^{15} + 279\,855z^{17} + 2\,706\,057z^{19} - 41\,340\,065z^{21} - 1\,026\,318\,519z^{23}$$
$$+ 1\,408\,558\,716z^{25} + 199\,683\,957\,550z^{27} - 179\,490\,745\,950z^{29}$$
$$- 31\,194\,220\,219\,944z^{31} + 212\,607\,938\,054\,812z^{33} + 2\,236\,204\,456\,367\,631\tfrac{3}{5}z^{35}$$
$$- 51\,766\,670\,838\,912\,054z^{37} + 493\,991\,498\,825\,723\,829z^{39}$$
$$- 3\,105\,393\,565\,906\,181\,508z^{41} + 14\,460\,086\,183\,397\,308\,820z^{43}$$
$$- 52\,552\,435\,320\,604\,611\,753\tfrac{1}{5}z^{45} + 153\,422\,998\,655\,275\,129\,635z^{47}$$
$$- 366\,107\,404\,100\,016\,173\,301z^{49} + 721\,734\,741\,868\,880\,994\,174z^{51}$$
$$- 1\,182\,631\,569\,859\,733\,767\,458z^{53} + 1\,614\,903\,212\,885\,941\,315\,542\tfrac{3}{5}z^{55}$$

$$-1\,836\,765\,168\,967\,899\,926\,238z^{57}+1\,733\,954\,085\,528\,545\,193\,990z^{59}$$
$$-1\,349\,310\,342\,861\,636\,166\,638z^{61}+856\,082\,771\,123\,121\,658\,342z^{63}$$
$$-435\,622\,654\,742\,070\,358\,693\tfrac{1}{5}z^{65}+173\,485\,867\,949\,262\,298\,242z^{67}$$
$$-52\,080\,426\,793\,957\,602\,030z^{69}+11\,081\,345\,228\,825\,091\,546z^{71}$$
$$-1\,489\,431\,081\,982\,439\,298z^{73}+95\,075\,403\,860\,118\,056\tfrac{6}{25}z^{75}$$

$$F_{11} = 12(18,9,3,6)+30(19,11,2,6)+9(20,13,1,6)-25(21,15,0,6)+12(17,6,6,5)$$
$$+189(18,8,5,5)+501(19,10,4,5)-651(20,12,3,5)-1\,458(21,14,2,5)$$
$$+795(22,16,1,5)+30(17,5,8,4)+573(18,7,7,4)+2\,025(19,9,6,4)$$
$$-4\,977(20,11,5,4)-21\,240(21,13,4,4)+10\,194(22,15,3,4)$$
$$+41\,031(23,17,2,4)-5\,730(24,19,1,4)-17\,088(25,21,0,4)$$
$$+9(17,4,10,3)+377(18,6,9,3)+4\,611(19,8,8,3)-5\,004(20,10,7,3)$$
$$-131\,284(21,12,6,3)+43\,866(22,14,5,3)+846\,507(23,16,4,3)$$
$$-561\,775(24,18,3,3)-1\,333\,173(25,20,2,3)+1\,354\,617(26,22,1,3)$$
$$-226\,105(27,24,0,3)+90(18,5,11,2)+3\,822(19,7,10,2)$$
$$+8\,796(20,9,9,2)-233\,802(21,11,8,2)-277\,311(22,13,7,2)$$
$$+4\,099\,560(23,15,6,2)-1\,578\,933(24,17,5,2)-20\,505\,333(25,19,4,2)$$
$$+27\,617\,778(26,21,3,2)+6\,887\,271(27,23,2,2)$$
$$-27\,285\,900(28,25,1,2)+11\,264\,358(29,27,0,2)$$
$$+6(18,4,13,1)+834(19,6,12,1)+11\,895(20,8,11,1)$$
$$-91\,479(21,10,10,1)-1\,099\,824(22,12,9,1)+5\,736\,510(23,14,8,1)$$
$$+15\,150\,240(24,16,7,1)-115\,676\,646(25,18,6,1)+112\,017\,072(26,20,5,1)$$
$$+430\,602\,573(27,22,4,1)-1\,271\,827\,071(28,24,3,1)$$
$$+1\,409\,229\,135(29,26,2,1)-732\,885\,294(30,28,1,1)$$
$$+148\,836\,996(31,30,0,1)+54(19,5,14)+1\,989(20,7,13)$$
$$+14\,741(21,9,12)-657\,720(22,11,11)+1\,282\,812(23,13,10)$$
$$+30\,633\,267(24,15,9)-168\,734\,358(25,17,8)+67\,256\,589(26,19,7)$$
$$+1\,881\,275\,672(27,21,6)-7\,149\,680\,127(28,23,5)$$
$$+13\,041\,321\,444(29,25,4)-13\,820\,971\,476(30,27,3)$$
$$+8\,680\,168\,152(31,29,2)-3\,004\,346\,190(32,31,1)+442\,432\,930\tfrac{1}{11}(33,33)$$

$$F_{12} = 1(19,9,3,7)+15(20,11,2,7)+6(21,13,1,7)+1(18,6,6,6)$$
$$+87(19,8,5,6)+345(20,10,4,6)+74(21,12,3,6)-732(22,14,2,6)$$
$$-441(23,16,1,6)+399\tfrac{1}{2}(24,18,0,6)+15(18,5,8,5)+345(19,7,7,5)$$
$$+2\,079(20,9,6,5)-201(21,11,5,5)-16\,206(22,13,4,5)$$

$-4\,481(23,15,3,5)+25\,710(24,17,2,5)-4\,740(25,19,1,5)$
$+708(26,21,0,5)+6(18,4,10,4)+386(19,6,9,4)+4\,413(20,8,8,4)$
$+3\,909(21,10,7,4)-86\,062(22,12,6,4)-109\,791(23,14,5,4)$
$+419\,107\tfrac{1}{2}(24,16,4,4)+321\,909(25,18,3,4)-645\,496\tfrac{1}{2}(26,20,2,4)$
$-217\,185(27,22,1,4)+281\,286\tfrac{1}{4}(28,24,0,4)+87(19,5,11,3)$
$+3\,687(20,7,10,3)+23\,166(21,9,9,3)-164\,208(22,11,8,3)$
$-817\,917(23,13,7,3)+2\,549\,566(24,15,6,3)+5\,417\,796(25,17,5,3)$
$-15\,430\,101(26,19,4,3)-4\,153\,653(27,21,3,3)+27\,455\,535(28,23,2,3)$
$-16\,103\,952(29,25,1,3)+1\,285\,087(30,27,0,3)+15(19,4,13,2)$
$+1\,148\tfrac{1}{2}(20,6,12,2)+23\,241(21,8,11,2)-69\,022\tfrac{1}{2}(22,10,10,2)$
$-1\,645\,523(23,12,9,2)+2\,904\,760\tfrac{1}{2}(24,14,8,2)+28\,911\,447(25,16,7,2)$
$-68\,391\,876(26,18,6,2)-109\,132\,098(27,20,5,2)$
$+426\,669\,024(28,22,4,2)-238\,802\,536(29,24,3,2)$
$-346\,707\,823\tfrac{1}{2}(30,26,2,2)+447\,773\,910(31,28,1,2)$
$-141\,596\,201(32,30,0,2)+93(20,5,14,1)+6\,450(21,7,13,1)$
$+37\,791(22,9,12,1)-962\,631(23,11,11,1)-3\,361\,056(24,13,10,1)$
$+52\,015\,392(25,15,9,1)-26\,866\,134(26,17,8,1)-803\,788\,821(27,19,7,1)$
$+2\,175\,490\,825(28,21,6,1)+698\,386\,962(29,23,5,1)$
$-11\,664\,438\,612(30,25,4,1)+21\,925\,272\,324(31,27,3,1)$
$-19\,445\,568\,651(32,29,2,1)+8\,646\,074\,205(33,31,1,1)$
$-1\,552\,275\,032(34,33,0,1)+6(20,4,16)+397(21,6,15)+12\,475\tfrac{1}{2}(22,8,14)$
$-61\,314(23,10,13)-3\,487\,058\tfrac{1}{4}(24,12,12)+23\,525\,820(25,14,11)$
$+93\,217\,965(26,16,10)-1\,208\,553\,569\tfrac{1}{3}(27,18,9)$
$+2\,896\,481\,885\tfrac{1}{4}(28,20,8)+6\,372\,449\,733(29,22,7)$
$-51\,951\,192\,909\tfrac{1}{2}(30,24,6)+136\,131\,584\,670(31,26,5)$
$-200\,407\,019\,946(32,28,4)+181\,344\,893\,268(33,30,3)$
$-100\,311\,816\,538\tfrac{1}{2}(34,32,2)+31\,215\,497\,907(35,34,1)$
$-4\,195\,534\,922(36,36).$

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